be of the complex form of the bracketed terms given in the example.

Consider any two such terms, symbolized by $X$ and $Y$. Evaluation of the model uses Eq. (14), repeated here for convenience; $P(X \cap Y) = P(X)\ P(Y)$. More accurate evaluation would be given by

$$P(X \cap Y) = P(X|Y)\ P(Y) \tag{18}$$

The use of Eq. (14) is always conservative, since component successes only are used in the reliability model, and

$$P(X|Y) \geqq P(X) \tag{19}$$

The magnitude of this error can also be approached from the unreliability model. This model includes the effects of certain failure patterns more than once. The patterns so redundantly included, however, are the intersections of more failures than are required to fail the system. With components of reasonable reliability, therefore, the numerical effect is small. The example illustrates this. Let

$$P[a_1] = P[a_2] = P[a_3] = P[a_4] = 0.10$$

Evaluation of the model for this example shows a reliability for the system >0.971 compared to a value of 0.972 from a Bayesian analysis of the system. With higher reliability components, the effect of statistical dependence would be even less noticeable. For example, if

$$P[a_1] = P[a_2] = P[a_3] = P[a_4] = 0.01$$

then the system reliability as given by the model is conservative by an additional increment of only $1.0 \times 10^{-6}$.

### Summary

The mathematical reliability model is developed from a readily available and philosophically conservative basis. The labor of model generation can be reduced to the preparation of computer input data in standard form. The output is easily interpreted; it can even be displayed as a series-parallel diagram. Evaluation is straightforward, eliminating any Bayesian analysis, and can also readily be performed on the computer. The two errors of approximation are small and tend to cancel each other. The unconservative error can be held below a specified upper bound.

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# Design of a Fairing for the Junction of Two Wings

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**A comparatively simple iterative method for the design of bullet-like fairings giving a subsonic, subcritical flow with a desired velocity at the junction of two wings is presented. It differs from published methods because its equations are consistently satisfied by using the calculated shape in the next calculation, until convergence is established. It can yield complete fairings, including sections at right angles to the freestream. The chief result is that further iterations may noticeably improve the shape. However, in all cases except one, convergence is well-advanced on the second iteration. The effect of slenderness on the waisting required is also illustrated. The validity of the method is partially established from independent theoretical and flight-test results. The report should be regarded as providing only part of the results, with further work capable of giving the section of the fairing at right angles to the freestream. This is tentatively assumed to be made up of circular arcs.**

## Nomenclature

$a$ = $x$ coordinate of the front of the fairing
$B$ = Prandtl-Glauert factor $[= (1 - M^2)^{1/2}]$
$b$ = $x$ coordinate of the rear of the fairing
$c$ = chord length of rectangular wing
$\pm F(x)$ = $z$ coordinate of the surface of the horizontal wing
$f$ = perturbation potential of the horizontal source sheet
$\pm G(x)$ = $y$ coordinate of the surface of the vertical wing
$g$ = perturbation potential of the vertical source sheet
$H(x,\theta)$ = $r$ coordinate of the surface including wings and fairing
$H(x,\theta^*)$ = $r$ coordinate of the design line
$H_1$ = $r$ coordinate of the design line at initial design point
$h$ = line source perturbation potential
$M$ = freestream Mach number
$q_H(x)$ = reduced line source strength
$V$ = velocity
$V_0$ = freestream velocity
$V_b(x)$ = velocity increment required on the design line
$x_1$ = $x$ coordinate of the initial design point
$(x, y, z)$ = Cartesian coordinates
$(x, r, \theta)$ = cylindrical coordinates
$\theta^*(x)$ = polar angle of the design line
$\phi$ = perturbation potential $(= f + g + h)$

***Subscripts***

$x,r,\theta$ = differentiation with respect to $x$, $r$, $\theta$

***Superscript***

$(\ )'$ = running coordinate

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## Introduction

THE use of bullet-like fairings to improve the flow at the junctions of intersecting wings is well-established. In published literature, an early subject is the reduction of the drag of struts.[1] Later, a vortex ring representation was used to design tail-junction fairings for shockless flow at subsonic speed.[2] A few remarks about this method are in order. Although the use of vortex rings is sound, the results obtained are poor, as is established from incompressible flow pressure distribution measurements. This is due chiefly to the use of only a few unknowns and results corresponding to only the first iteration used here. The method does not lend itself

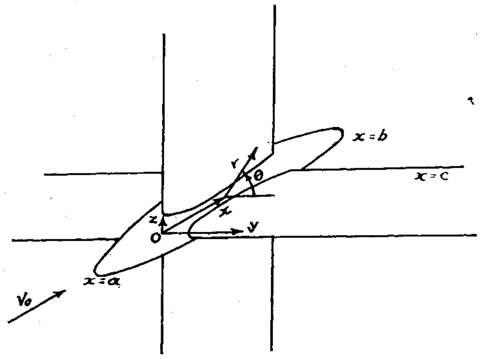


**Fig. 1 General arrangement of the wings and fairing.**

easily to an increase in the unknowns and to iteration, in which the calculated shapes are used in successive calculations, because the vortex-ring-induced velocities are elliptic integrals.[3] The results of slender-body theory would also correspond to the first iteration only, to which is added the slenderness restriction.

In the subject method, a line source and intersecting source sheets are used to represent a bullet-like fairing at the junction of two doubly infinite cylindrical wings, with symmetrical sections, at right angles to each other, and at zero incidence. This enables the fairing to be designed for a subsonic, subcritical flow, with a desired velocity at the junction. The method is easily extended to deal with finite wings, while linear theory and other approximations enable the configuration assumed here to be applied to more practical cases.[4] It differs from the vortex ring method and slender-body theory because its equations are consistently satisfied by using the calculated shape in the next calculation, until convergence is established. The method lends itself to this because the induced velocities are elementary functions. It can yield complete fairings, including sections at right angles to the freestream, whereas this problem is not dealt with in the vortex ring method.

The method is applied to identical joined wings. Different initially assumed fairing ordinates are used to determine their effect on the convergence of the shapes. To establish the validity of the method, the linear-theory velocity induced by the corrugated quasi-cylinder without wings is specified, and the resulting design is compared with this shape. Flight tests are used to verify a design placed on a T tail and obtained with a combination of this method and its inverse, that is, one which gives the velocities on a given fairing.

The report should be regarded as providing only part of the results, with further work capable of giving the section of the fairing at right angles to the freestream. This is tentatively assumed to be made up of circular arcs.

## Theory

Let $f$, $g$, and $h$ be the potentials induced by doubly infinite source sheets in $z = 0$ and $y = 0$ and by a line source on the $x$ axis (Fig. 1). Then

$$\phi = f + g + h \tag{1}$$

is the perturbation potential. Further, let

$$\frac{f}{V_0} = \frac{1}{2\pi B}\int_0^c F_{x'}\cdot\log_e[(x - x')^2 + (Br\sin\theta)^2]dx' \tag{2}$$

$$\frac{g}{V_0} = \frac{1}{2\pi B}\int_0^c G_{x'}\cdot\log_e[(x - x')^2 + (Br\cos\theta)^2]dx' \tag{3}$$

$$\frac{h}{V_0} = \frac{-1}{4\pi B^2}\int_a^b \frac{q_H(x')dx'}{[(x - x')^2 + (Br)^2]^{1/2}} \tag{4}$$

where $B$ is the Prandtl-Glauert factor. These particular sources enable the configuration to be represented with the degree of freedom required for the determination of the fairing shape. Alone, the source sheets contribute the potential, according to linear theory, for doubly infinite cylindrical wings, whose surfaces are $z = \pm F(x)$ and $y = \pm G(x)$. The line source of reduced strength $q_H(x)$ adds a fairing to the junction, which is otherwise unspecified, to allow it to be determined from the other conditions, described below.

In cylindrical coordinates, the equation of the surface is

$$r - H(x,\theta) = 0 \tag{5}$$

The condition that the velocity normal to the surface is zero reduces to

$$H_x = \frac{\phi_r}{V_0 + \phi_x} - \frac{1}{V_0 + \phi_x}\cdot\frac{\phi_\theta}{H}\cdot\frac{H_\theta}{H} \tag{6}$$

where $V_0$ is the freestream velocity, parallel to the $x$ axis. This becomes

$$H_x = \phi_r/(V_0 + \phi_x) \tag{7}$$

where

$$\phi_\theta = 0 \tag{8}$$

This position is called the design line and its coordinates are $x$, $\theta^*(x)$, and $H(x,\theta^*)$. If $H_\theta\cdot(d\theta/dx)$ is neglected in comparison with $H_x$, integration along the design line yields approximately

$$H = H_1 + \int_{x_1}^{x} \frac{\phi_{r'}}{V_0 + \phi_{x'}}\,dx' \tag{9}$$

where $H_1$ is the radial coordinate of the design line at the initial design point $x_1$, unchanging with iteration. The velocity ratio on the design line is

$$V/V_0 = 1 + (\phi_x/V_0) + \tfrac{1}{2}\cdot(\phi_r/V_0)^2 \tag{10}$$

approximated to the second order of small quantities. It is also required to be

$$\frac{V}{V_0} = 1 + \frac{V_b(x)}{V_0} + \frac{1}{2}\cdot\left(\frac{f_x + g_x}{V_0}\right) \tag{11}$$

This is not a derived result but a desired condition to be achieved at the junction to reduce its velocity. For $V_b(x) \equiv 0$ and on the design line, at least, it insures a reduction in the velocity increment to only half that induced by the wings alone, thereby approximating wing conditions far away from the junction. The velocity increment $V_b(x)$ is chosen to satisfy other requirements, but is otherwise freely disposed. For example, the junction velocity increment may be allowed to exceed $(f_x + g_x)/2V_0$, to reduce the size of the needed fairing, or $V_b(x)$ may be used to design axisymmetric bodies without wings. Thus $V_b(x)$ provides a more flexible specification. The equating of (10) and (11) yields

$$\frac{h_x}{V_0} = \frac{V_b(x)}{V_0} - \frac{1}{2}\left(\frac{f_x + g_x}{V_0}\right) - \frac{1}{2}\left(\frac{f_r + g_r + h_r}{V_0}\right)^2 \tag{12}$$

If the expressions for the source potentials are substituted, this is an integral equation, whose unknowns are $\theta^*$, $H(x,\theta^*)$, and the reduced line source strength, $q_H(x)$. The required three equations are completed by (8) and (9).

The solution is iterative. To obtain a tractable approximate set of equations,[4] integrals $h_x$, $f_x$, $g_x$, $h_r$, $f_r$, and $g_r$ are expressed as sums of elementary integrals, by approximating the source distributions by elementary functions, piecewise continuous, which contain disposable coefficients. The wing source coefficients are obtained in terms of the given ordinates of the wings. Those for the line source are unknowns. Initially $h_r$ is equated to zero and suitable distributions of $\theta^*(x)$ and $H(x,\theta^*)$ are assumed. Equation (12) then yields $h_x$ explicitly. Its substitution in

$$\frac{1}{4\pi B^2}\int_a^b \frac{(x-x')q_H(x')dx'}{[(x-x')^2+(BH)^2]^{3/2}} = h_x \qquad (13)$$

which is obtained from (4) by differentiation, and use of the approximate form of its inverse,[4] which is a set of ordinary linear simultaneous equations, yields the line source coefficients. Consequently, all the potential derivatives are known and evaluation of (9) (by the trapezoidal rule) yields $H$. The function $\theta^*(x)$ is found by systematically varying $\theta$ for each $x$, until (8) is satisfied.

The calculation is repeated from (12) down, with the new $h_r$, which is found explicitly in terms of the now known $q_H(x)$, and with the new $\theta^*$ and $H(x,\theta^*)$, until convergence establishes that the complete set of equations is consistently satisfied.

If the first and last design points are sufficiently far from the wings, the regions beyond these points may be replaced by round-nose or other axisymmetric forms whose sections in streamwise planes need only to be smoothly faired into the rest of the shape. This is because the potential reduces to that induced by the line source only and because the contributions from here to the velocity in the design region are small. Reference 4 describes a better approximation for the nose-induced velocities. In principle, once the nose region and line source are known, the full shape of the fairing can be determined by integration of the boundary condition, (6), to obtain the section at right angles to the freestream. This shape would then be justified by showing that its velocities were acceptable.

As is known, the potential derivatives are not valid near round leading edges and round fairing noses. To prevent the design line from being thus compromised, the pivotal points are kept away from these places. This can be achieved by suitably disposing the nose and tail of the fairing and by altering the number of pivotal points. This partly accounts for the use of derivatives of (2) and (3) which do not use an $r = 0$ approximation.[4]

To explain why it is preferred to deal with $q_H(x)$ directly as an unknown, it may be shown by means of the expression for $h_r$ and (7) approximated to the first order that

$$\frac{q_H(x)}{V_0} = 2\pi B^2 H\left[H_x - \left(\frac{f_r+g_r}{V_0}\right)\right] \qquad (14)$$

with the right-hand-side quantities evaluated on the design line. In the design problem, it is relatively inconvenient to use (14). However, if this geometry is specified, (14) explicitly relates $q_H(x)$ to known quantities, thereby enabling the velocities to be evaluated without matrix inversion and by means of quadratures.[5]

It is appreciated that the method is mathematically inconsistent. For example, (7) and (10) might imply a greater accuracy than is possible, in view of the errors inherent in the linear theory solutions, (2–4). It was expedient to accept these inconsistencies on the grounds that no changes are required to give an increased order of accuracy when the method is applied to bodies of revolution in incompressible flow. For this reason, (10) might well be replaced by the exact expression for $V/V_0$ from which it is derived, although (10) is a little more convenient to use. Equation (10) is also consistent with the well-established pressure coefficient formula applicable to bodies of revolution in compressible flow.

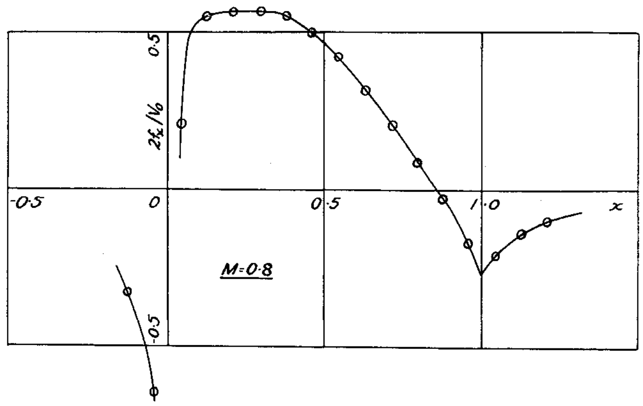


**Fig. 3 Wing-induced velocity increment at the junction.**

The method is readily extended to other wing configurations. The assumptions that the wing extremities coincide at the junction and that the wings are doubly infinite, although convenient, are unnecessary. When these are relaxed, the equations still apply to arbitrary symmetrical planform wings of finite aspect ratio, if the appropriate expressions for the wing source potentials are used. In particular, swept wings may be dealt with, although in this case it may be more convenient to approximate the wing-induced velocities by their values on the $x$ axis ($r = 0$), perhaps suitably modified to make them uniformly valid. On the other hand, linear-theory approximations and others enable the results for doubly infinite unswept wings to be applied to wings at incidence and to symmetrical and unsymmetrical wings of finite aspect ratio, more representative of practical empennages.[4] Even swept wings are amenable to treatment with the assumed configuration by means of suitable changes in the specified velocity, $V_b(x)$, and wing ordinates.

## Results

The simplification of identical wings is used. For this case, $\theta^* = \pi/4$, which eliminates the need for (8), while (9) holds exactly. Also, $V_b(x) \equiv 0$, and the section of the fairing at right angles to the freestream is assumed to be made up of circular arcs.

Figure 2 illustrates the disagreement between the final and calculated ordinates at the first iteration, for the junction velocity illustrated in Fig. 3, obtained for 12% thickness-chord ratio wing sections, occupying $x = 0$ to $x = 1$. Linearly increasing initially assumed ordinates improve the comparison (Fig. 2). However, convergence is well-ad-

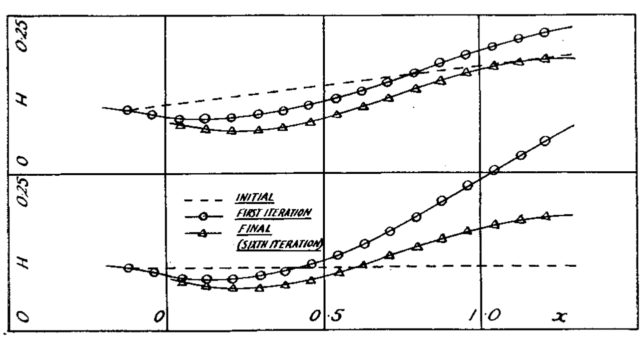


**Fig. 2 Effect of initially assumed ordinates on convergence at first interation.**

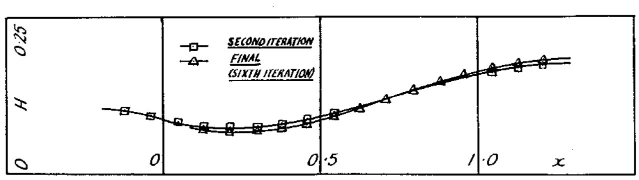


**Fig. 4 Convergence of second iteration ordinates.**

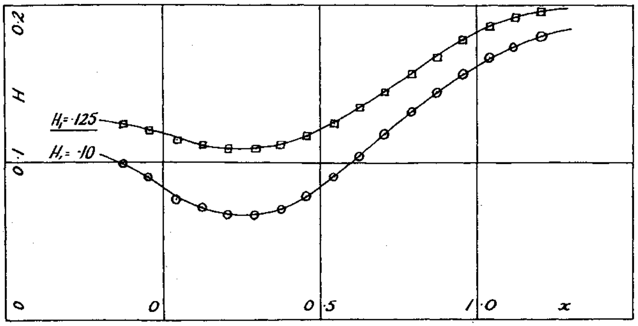


**Fig. 5 Effect of slenderness on waisting required.**

vanced on the second iteration (Fig. 4). This is typical of all cases, except that for a linearly decreasing set of ordinates, which did not converge.[4] As expected, the waisting is increased by a decrease in the width of the body, as is shown by the values of the parameter $H_1$ (Fig. 5).

To establish partially the validity of the method, the streamwise velocity increment obtained from linear (not slender-body) theory for a corrugated quasi-cylinder is specified. Figure 6 shows that the calculated slope distribution has the same form but that it is translated down. The addition of a source of uniform strength to the region designed brings the calculated slopes and ordinates (Fig. 7) into excellent agreement, while causing a very small change in the streamwise velocity increment $h_x$ from that specified for the design. That this is justified is inferred from the properties of this line source. The full set of equations is thereby satisfied approximately.

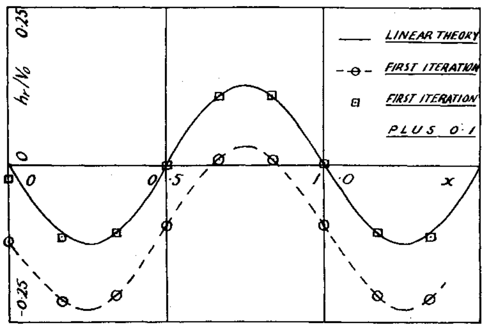


**Fig. 6 Slope distribution—corrugated cylinder.**

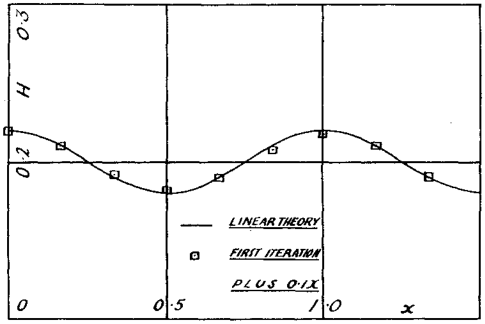


**Fig. 7 Ordinate distribution—corrugated cylinder.**

There is a rather indirect check from flight tests. The design has sections made up of circular arcs. It is placed on a T tail and it is obtained by means of a combination of this method and its inverse.[5] The adequacy of the fairing is justified by the behavior of the aircraft, which shows no evidence, even at $M = 0.85$, of a shock-induced pitch effect, previously observed at $M = 0.78$.

## Conclusions

Noticeable shape improvements may result from the designing of bullet-like fairings for desired velocities by means of a method which consistently satisfies the equations. It is noted that in all cases, except one, convergence is well-advanced on the second iteration.

The validity of the method is established partially by showing that the corrugated quasi-cylinder is retrieved when its linear-theory velocity is specified. In flight tests, the adequacy of the fairing is justified by the elimination of a previously observed shock-induced pitch effect.

## References

[1] Thwaites, B., *Incompressible Aerodynamics* (Oxford University Press, Oxford, England, 1960), Chap. X, p. 437.

[2] Kuchemann, D. and Weber, J., "Low speed tunnel tests on intersection of two rectangular wings with various fairings at the junction," Royal Aircraft Establishment Rept. Aero 2260 (May 1948).

[3] Kuchemann, D. and Weber, J., *Aerodynamics of Propulsion* (McGraw-Hill Book Company Inc., New York, 1953), Appendix.

[4] Sopher, R., "Fairing for the junction of two wings," Canadair Rept. ERR-CL-RAA-00-116 (July 1965).

[5] Sopher, R., "Bullet-like fairing for the tail-junction of the CL-41G," Canadair Aerodynamic Memorandum AM-41-129 (August 1965).